Parametric characterization of the geometry of honed cutting edges

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Abstract

Development of methodologies for the geometric characterization of cutting edges is of significant current interest, in light of the profound influence that the edge geometry wields on virtually every machining response, and the evolving capability for generating tailored edges. This paper proposes the parametric modeling of the tool edge geometry through the application of free-knot B-splines that comprise three piecewise segments corresponding to the cutting edge profile and the two tool faces. The transition points that demarcate the cutting edge from the tool faces are objectively and robustly identified by the adaptive placement of the knots that minimizes the residual error from fitting the B-spline to the tool profile data. On identification of the cutting edge, the edge profile is modeled by parametric quadratics to yield four geometrically-relevant, contour-based parameters that characterize both symmetric and asymmetric honed edges.

Key words: B-spline, cutting edge, edge preparation, machining

1 Introduction

The role of the meso-geometry of the cutting edge on the mechanics of chip formation was first documented by Albrecht [1] in 1960. It has however taken several decades since, for this important aspect to be accorded its due consideration. The geometry of the edge has now been demonstrated to affect practically every machining response including: forces [2], temperature distribution [3], tool life [4], surface finish [5] and residual stress [6], in addition to

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wielding a controlling influence on process stability [7]. Consequently, there is much interest of late amongst the machining research community on the generation and characterization of the cutting edge.

Conventional edge preparation techniques such as brush honing and micro-blasting correspond to significant variability in edge geometry, not just between edges but also along the profile of the same edge [8]. This is of particular detriment in the machining of high-value components in terms of robust tool performance. Avenues to addressing this issue have involved process developments such as 5-axis brush honing [9], and the conception of novel processes that entail magneto-abrasive [10], laser [11] and electro erosion [12] techniques. Further to the variability arising from the process itself, it is intriguing to note that the lack of a standard characterization methodology is also a factor contributing to variability, as identified in [13].

In terms of edge characterization, it has thus far been expedient in most instances to specify the geometry of honed edges by a simple edge radius parameter. This inherently assumes the edge profile that bridges the rake and flank faces to conform to an arc of a circle. This need not however be the case, and indeed tool performance may be enhanced by rendering the cutting edge to be appropriately asymmetric [3]. Given the critical influence of edge geometric attributes on process responses, and the evolving capability of aforementioned novel processes in the generation of tailored cutting edges, it is imperative to develop methodologies for the comprehensive geometric characterization of the cutting edge.

Denkena et al. [3] proposed that the edge geometry be characterized with reference to the virtual tool tip derived from the linear extension of flank and rake faces (Fig. 1a). Any asymmetry in the edge geometry is signified by the ratio of distances S_α and S_γ from the tool tip to points 1 and 2, from where the edge profile diverges away from the flank and rake faces, respectively. The degree of edge flattening is specified by parameter Δr which is the distance from the virtual tool tip to the apex of the edge profile, and parameter φ locates the tool apex relative to the tool faces. Such a characterization is simple and facilitates easy visualization of the edge; however, the said parameters are evaluated based on just three points on the edge profile, which are not adequate to uniquely characterize the edge geometry. Furthermore, ref. [3] did not specify a method to objectively determine the transition points 1 and 2

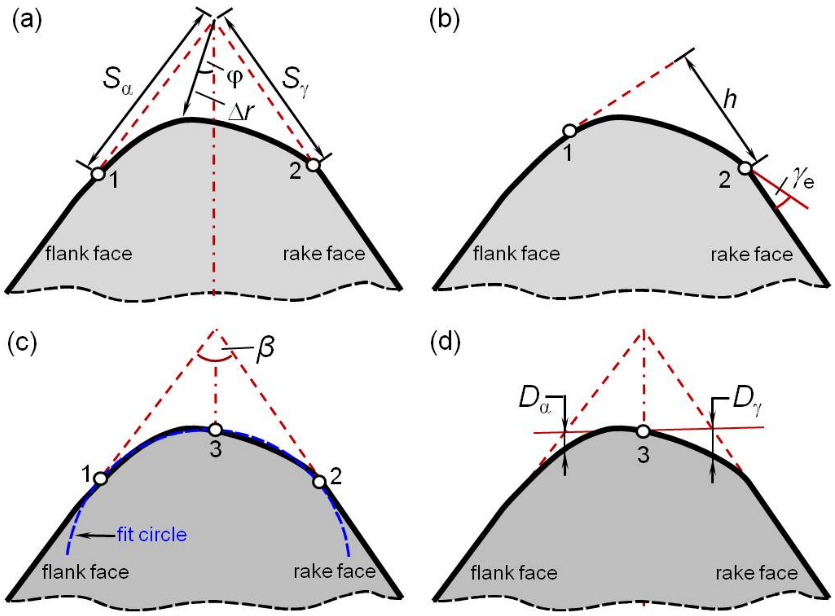

Fig. 1. Edge characterization methods due to: (a) Denkena et al. [3], (b) Rodriguez [14], and (c & d) Wyen et al. [15].

wherefrom the edge profile deviates off the tool faces.

Rodriguez [14] attempted to address the issue above by defining the transition points 1 and 2 in terms of the intersection of the edge profile (Fig. 1b) with the uncut chip thickness h, with the orientation of the edge specified with respect to the effective rake angle γ_e. This renders the edge geometric parameters to be dependent on the cutting conditions rather than be intrinsic to the cutting edge. Rodriguez further approximated the edge profile by a sixth-degree polynomial, with the unfavorable implication [15] that the error between the edge profile and the interpolating polynomial of such high order could be unacceptably high, due to oscillation effects known as Runge's phenomenon in numerical analysis.

Wyen et al. [15] recently proposed a method for identifying the transition points that delineate the meso-geometry of the cutting edge from the flank and rake faces of the tool. The first step in this iterative method involves least squares fitting of straight lines to represent the tool faces over a domain that spans a certain preset distance from the apex of the edge profile. These lines are extended to locate the virtual tool tip to enable the computation of the point of intersection of the edge profile and the bisector of the wedge angle β (point 3 in Fig. 1c). A circle is then constructed such that it passes through point 3 and is tangential to the said straight lines representing the tool faces. Points 1 and 2 at which this circle intersects the straight lines are considered first approximations for the transition points, which constitute the updated upper limit for the subsequent least squares fitting of the tool faces. The steps above are iterated to refine the location of the transition points.

As the edge is approximated as a circle in this method, any deviations from the ideal such as the edge being asymmetric adversely affects edge identification. In consideration of this, Wyen et al. [15] proposed that the fit be reconsidered should the value of the coefficient of determination R^2 be less than a predefined value, say 90%. They further proposed that any cutting edge asymmetry be denoted by the ratio of distances D_α and D_γ measured from a line passing through point 3 and perpendicular to the wedge angle bisector, to the edge profile, as shown in Fig. 1d. This is different from the proposal in [3] in that the distances D_α and D_γ are largely independent of the transition points.

The review of the relevant literature above clearly indicates that a novel approach is required for the identification and characterization of the cutting edge, given that published methods refer to ideal geometric shapes and to discrete points on the edge profile. In this context, Section 2 of this paper proposes the application of parametric B-splines for the contour-based identification of the transition points that demarcate the cutting edge from the tool faces. Being piecewise polynomials, B-splines are a logical choice for such identification, as the flank/rake faces and the edge profile that constitute the cutting edge are themselves piecewise in nature. The robustness of such an identification method is investigated in light of several potential sources of uncertainties. Following edge identification, the characterization of the cutting edge profile using parametric quadratics is proposed in Section 3. Four contour-based parameters are derived to uniquely quantify the geometry of a edge, which are exemplified in terms of their application to both symmetric and asymmetric honed edges.

2 Cutting edge identification

As the meso-geometry of a tool edge profile comprises two tool faces that flank the cutting edge (Fig. 2), unambiguous identification of the transition points that separate the edge from the tool faces is the first step that should precede edge characterization. The critical importance of this is illustrated in

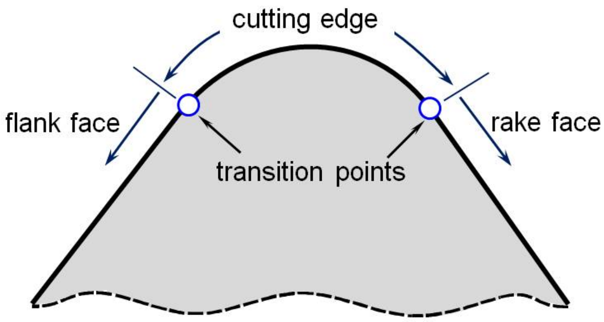

Fig. 2. Terminology of tool edge profile.

Fig. 3, wherein it is shown that the same symmetrically honed edge could correspond to significantly different edge radius (r_β) values, depending on where the color-coded sets of transition points are deemed to be located. The uncertainty imposed by the boundaries of the cutting edge domain necessitates an edge identification method capable of uniquely and objectively demarcating the cutting edge from the rake and flank faces.

The key idea in the proposed contour-based edge identification method is that the edge geometry may be modeled by three piecewise segments to represent the flank/rake faces and the cutting edge, with zero and first geometric continuity at the joining points. Multi-degree splines are most suited to this end, with the first and third sections corresponding to flank and rake faces constrained to be linear, while assuming higher orders for the curvilinear cutting edge in between. As the approximation algorithms for such curves are somewhat complex, a simpler alternative is to employ a single-degree piecewise curve, wherein the first and third segments are geometrically constrained to conform to the linear shape of the tool faces. The profile of the cutting edge is usually free of any change in the direction of curvature, and hence a second

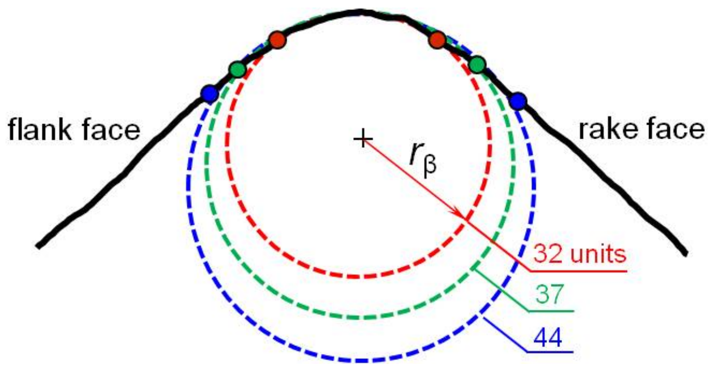

Fig. 3. Effect of deemed edge boundary on edge radius r_β.

degree curve should suffice for modeling it.

In terms of cutting edge identification, the joining points of the spline sections assume great importance as they correspond to the transition points that demarcate the cutting edge from the tool faces at either ends. With this in view, an appropriate family of spline curves would be the one that enables the curve fitting process to be optimized with respect to the points that connect the three segments. A family of curves that offers this feature are piecewise parametric polynomials called B-splines [16], which are defined as:

$$\mathbf{C}(u) = \sum_{i=0}^{n} N_{i,p}(u)\mathbf{P}_i \tag{1}$$

where $u \in [0, 1]$, and $\mathbf{P}_i$ are $n + 1$ control points that represent the polyline form of the curve in a way that the actual curve always lies completely in their convex hull. $N_{i,p}(u)$ are B-spline basis functions of degree p computed recursively as:

$$\begin{aligned} N_{i,0}(u) &= \begin{cases} 1 & \text{if } u_i \le u \le u_{i+1} \\ 0 & \text{otherwise} \end{cases} \\ N_{i,p}(u) &= \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u) \end{aligned} \tag{2}$$

Eqn. (1) can be expressed as:

$$C = NP \tag{3}$$

where N and P are matrices of basis function coefficients and control points, respectively.

Basis functions facilitate the piecewise nature of B-spline curves by making use of parametric values called knots (u_i). Each u_i is one of $(m + 1)$ B-spline knots from the knot vector $[u_0, u_1, \ldots, u_m]$ and corresponds to a parameter at which one polynomial section ends and another begins. In other words, knots may be considered as division points that subdivide the curve into segments with $(p - 1)$ continuity at the joining points. For B-spline curves, the knot vector is usually on the $[0, 1]$ interval, with the $(p + 1)$ first and last knots being identical, as in:

$$
\begin{aligned}
[u_0 = u_1 = \ldots = u_p = 0, u_{p+2}, \ldots, u_{m-p-1}, u_{m-p} = u_{m-p+1} \\
= \ldots = u_{m-1} = u_m = 1]
\end{aligned}
\tag{4}
$$

The number of knots $(m+1)$, the degree of the B-spline curve p and the number of control points $(n+1)$ are related as:

$$
m = p + n + 1 \tag{5}
$$

As indicated previously, the proposed method uses a quadratic B-spline to model the cutting edge. For a second degree B-spline to constitute three polynomial sections that correspond to the flank/rake faces and the cutting edge, the knot vector should have three spans and will be of the form $[0, 0, 0, u_1, u_2, 1, 1, 1]$. Based on this knot vector, the second degree basis function matrix N can be computed from eqn. (2) to be:

$$
\begin{bmatrix}
\frac{(u_1-u)^2}{u_1^2} & -\frac{u^2}{u_1^2}+\frac{2u}{u_1}-\frac{u^2}{u_1u_2} & \frac{u^2}{u_1u_2} & 0 & 0 \\
0 & \frac{(u_2-u)^2}{u_2(u_2-u_1)} & \frac{(u^2-u_2)u_1-u^2+2uu_2-u_2u^2-u_2u^2}{u_2(u_2-u_1)(1-u_1)} & \frac{(u-u_1)^2}{(1-u_1)(u_2-u_1)} & 0 \\
0 & 0 & \frac{(1-u^2)}{(1-u_1)(1-u_2)} & \frac{(1-u^2)(u-u_1)}{(1-u_1)(1-u_2)}+\frac{(1-u)(u-u_2)}{(1-u_2)} & \frac{(u-u_2)^2}{(1-u_2)^2}
\end{bmatrix}
\tag{6}
$$

For a second degree polynomial $(p = 2)$ with eight knots $(m + 1 = 8)$, five control points $(n + 1 = 5)$ are required to uniquely define the curve, as per eqn. 5. Knowing u_1,u_2 and five control points, the desired B-spline curve $C(u)$ is fully defined using eqn. (3) and eqn. (6), and the cutting edge will be subdivided into three quadratic profiles joining smoothly at u_1 and u_2, as shown in Fig. 4. The equation for each individual section of the cutting edge profile can be derived from the corresponding row of the N matrix (eqn. 6) multiplied by the control points matrix P.

The profile of a tool edge can be acquired by techniques such as profilometry, white light interferometry and confocal microscopy. With the profile data points $\mathbf{D} = [D_1, \ldots, D_q]$ in place, the edge identification algorithm reduces to the determination of u_1 and u_2. However, before proceeding with such fitting computations, a set of parameters $t = [t_1, \ldots, t_q]$ should initially be assigned to the data points so that $C(t_i) \approx D_i$ holds subsequent to fitting, a process known as parametrization. Among the different methods available for

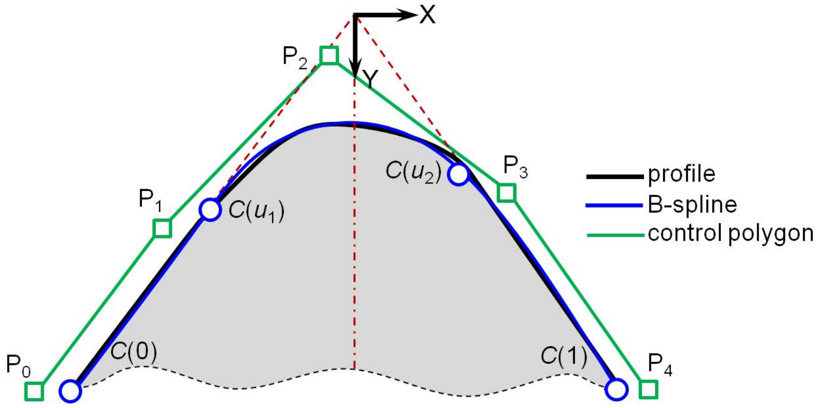

Fig. 4. Edge identification using a quadratic B-spline with three sections.

parametrization, the method used in this research refers to uniformly spaced parameters:

$$t_i = \frac{i-1}{q};\ i = 1 : q \tag{7}$$

The algorithm for finding the cutting edge profile is then equivalent to the minimization of the least square B-spline curve fitting function:

$$L = \sum_{i=1}^{q} |C(t_i) - D_i|^2 \tag{8}$$

The unknowns are five control points ($n + 1 = 5$), u_1 and u_2.

If the values of u_1 and u_2 are predetermined (as in uniformly spaced knot generation, e.g. $u_1 = 1/3$ and $u_2 = 2/3$), the problem simplifies to a fixed knot B-spline approximation, and eqn. (8) will reduce to a linear least square problem that can be solved by simple linear algebraic formulations. However, with u_1 and u_2 unknown, the fit is allowed to modify the boundaries of sections for the best-fit three-segment quadratic polynomial. This is a key point in that the transition points that demarcate the cutting edge from the tool faces are objectively and robustly identified by the adaptive placement of the knots that minimizes the residual error from fitting the B-spline to the tool profile data. This type of B-spline curve fitting is categorized as free-knot B-spline approximation, and as a result eqn. (8) will be a nonlinear least squares problem, which can be solved by iterative numerical methods like Gauss-Newton [17].

To render the first and third segments corresponding to flank and rake faces to conform to the geometry of the tool faces, the approximation procedure is further coupled with two geometric constraints:

$$\begin{aligned} &\frac{dC_y}{dC_x}(0) = m_1 \\ &\frac{dC_y}{dC_x}(1) = m_2 \end{aligned} \tag{9}$$

where m_1 and m_2 are the slopes of the lines representing the flank and rake faces.

In the context of the background above, the steps to cutting edge identification are enumerated below and accordingly illustrated in Fig. 5, with the virtual tool tip as the origin of the coordinate system.

(1) Having the profile data points ($\mathbf{D}_0$), two horizontal margins that best include the linear sections on rake and flank faces are given as inputs. Based on linear regression of the points contained between these margins, the rake and flank lines, and the origin of the coordinate system are constituted.
(2) Using these rake and flank lines, the wedge angle β is computed. If the

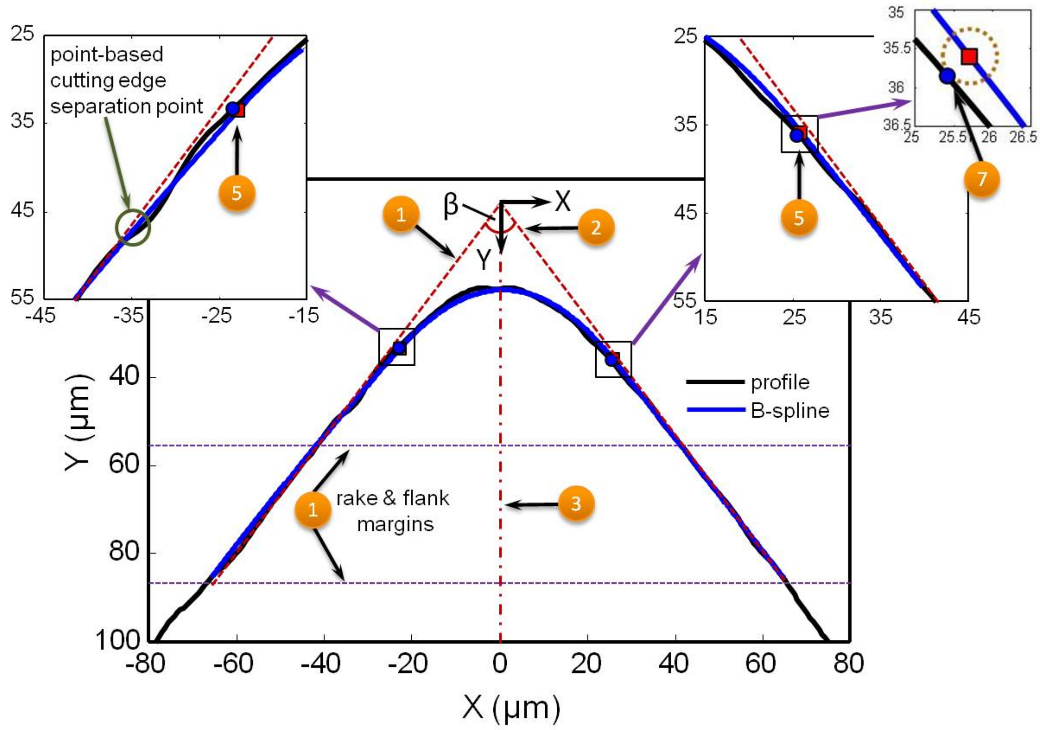

Fig. 5. Cutting edge identification by free-knot B-spline approximation.

computed wedge angle is not within a predefined margin (say $\pm 5\%$) of the nominal value, step one is repeated with two updated horizontal marginal lines.

(3) Profile data points are then rotated ($\mathbf{D}$) such that the wedge angle bisector coincides with the Y axis, for which the rake and flank line slopes (m_1 and m_2) will be equal but with opposite signs.

(4) Profile data points are then parameterized using eqn. (7).

(5) Eqn. (8) is solved for profile points ($\mathbf{D}$) starting from the lower marginal line using Gauss-Newton [17] method, with eqn. (9) as constraints. The resultant u_1 and u_2 will provide the first guess for the transition points, with the cutting edge bounded between $C(u_1)$ and $C(u_2)$.

(6) If the closest distance of $C(u_1)$ (or $C(u_2)$) to flank (or rake) line is more than a threshold value (say 3% of $|C(u_2) - C(u_1)|$), u_1 (or u_2) will be decreased (or increased) until this condition is met.

(7) The closest points on the actual profile ($\mathbf{D}$) to $C(u_1)$ and $C(u_2)$ will be the start and end points of the edge region.

The algorithm above is applicable to all edge geometries; however, if the cutting edge profile contains an abrupt slope change between the sections that the quadratic polynomial cannot closely follow, the precise identification of the cutting edge is affected. One example of such abrupt geometric change is a T-land (chamfer) edge geometry for which all three tool sections are linear. It is suggested that first degree B-splines rather than quadratics be used in such an instance. All other steps remain unaffected.

Point-based cutting edge identification methods which consider the edge boundaries to be defined by the points from where the edge profile separates off the rake and flank lines are quite sensitive to minor form errors and profile irregularities. This issue is addressed by the proposed contour-based method, as illustrated in the inset on the left of Fig. 5, wherein the transition point can be seen to be not affected by variations in the form of the tool profile.

The proposed algorithm is intended to minimize the influence of potential sources of uncertainty in cutting edge identification. This algorithm is based on the fact that rake and flank geometries can be rendered as two lines. Inappropriate consideration of the tool macro-geometry comprising the rake and flank faces might result in improper cutting edge identification. As shown in Fig. 6, points lying between horizontal margins 1 and 2 contain the actual

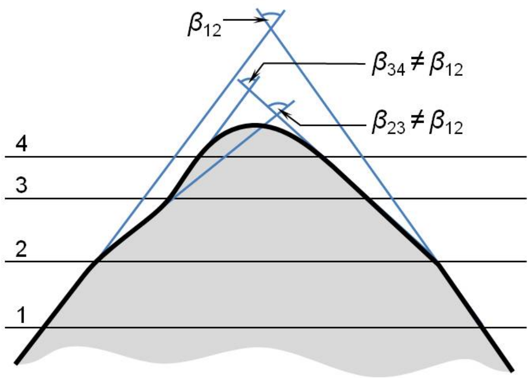

Fig. 6. Identification of tool face margins in consideration of wedge angle β.

macro-geometry of the cutting tool, referring to the nominal wedge angle β_{12}. Erroneous consideration of points between margins 2 and 3, or 3 and 4, results in entirely different flank and rake lines that respectively refer to wedge angles β_{23} and β_{34} that are different from the nominal value. The comparison of the computed tool wedge angle with the nominal value in the second step of the identification algorithm precludes such misinterpretations. Step 6, though rarely required, is added to ensure that the edge is properly determined.

Fig. 7 depicts three edge profiles with a point uncertainty of 1 μm to demonstrate the robustness of the edge identification method against such. The variability in the location of the edge identification points is less than 2 μm, which highlights the superiority of a contour-based approach. One of the sources of uncertainties in the identification of the cutting edge refers to the distance

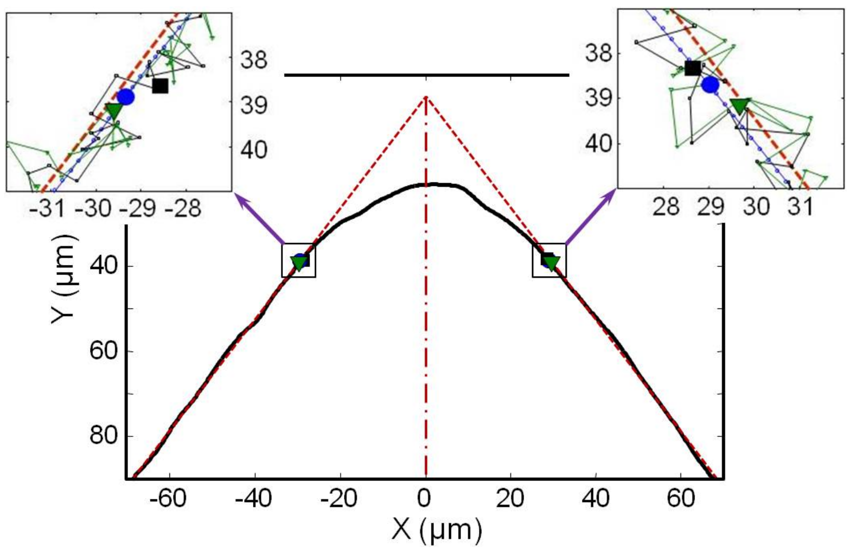

Fig. 7. Robustness of edge identification against point uncertainty.

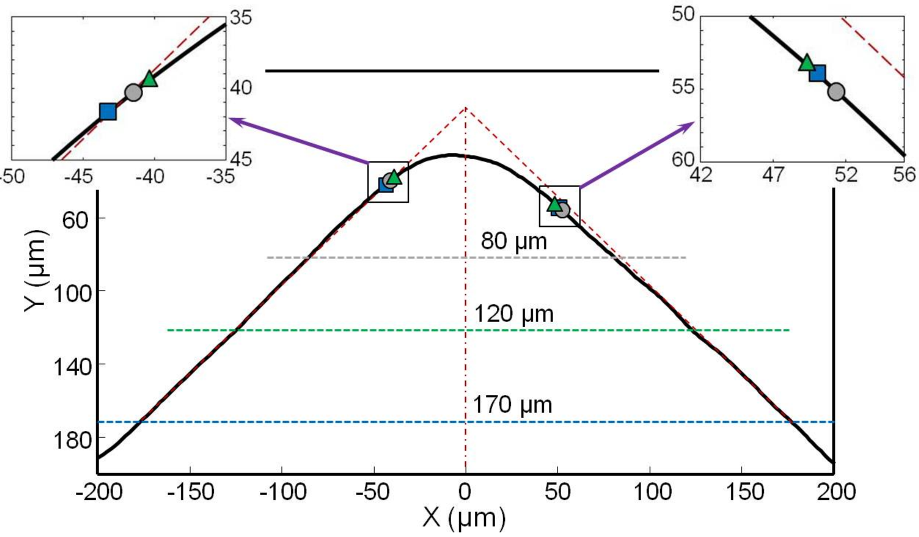

Fig. 8. Invariability of edge identification algorithm to tool face region considered.

from the tool tip over which rake/flank faces are being considered for modeling. While unreasonably too big or too small values of this distance will lead to inaccurate edge identification, any algorithm should yet be as invariable as possible to this factor. Fig. 8 illustrates the robustness of the proposed edge identification method against the marginal distance (lower margin in Fig. 5) by taking advantage of free-knot B-spline approximation. Intentional translation of the lower marginal line from 80 μm up to 170 μm can be seen to induce a variability of less than 2 μm in the identification of the cutting edge boundaries.

3 Cutting edge characterization

As reviewed in section 1, methods currently used to characterize edge geometry refer to parameters that are evaluated based on discrete points on the edge profile. Fig. 9 illustrates issues arising from such an approach. Fig. 9a shows two edges that would refer to different machining responses to correspond to identical S_α and S_γ parameters used in ref. [3]; similarly, Fig. 9b illustrates the limitation of parameters D_α and D_γ proposed in ref. [15] in being able to effectively distinguish the edges, as the two entirely different profiles correspond to identical parameters.

To characterize an edge by a comprehensive set of contour-based parameters, it may be modeled by a geometric curve that can be adequately fit to various edge geometries. To this end, it is proposed herein that parametric quadratic

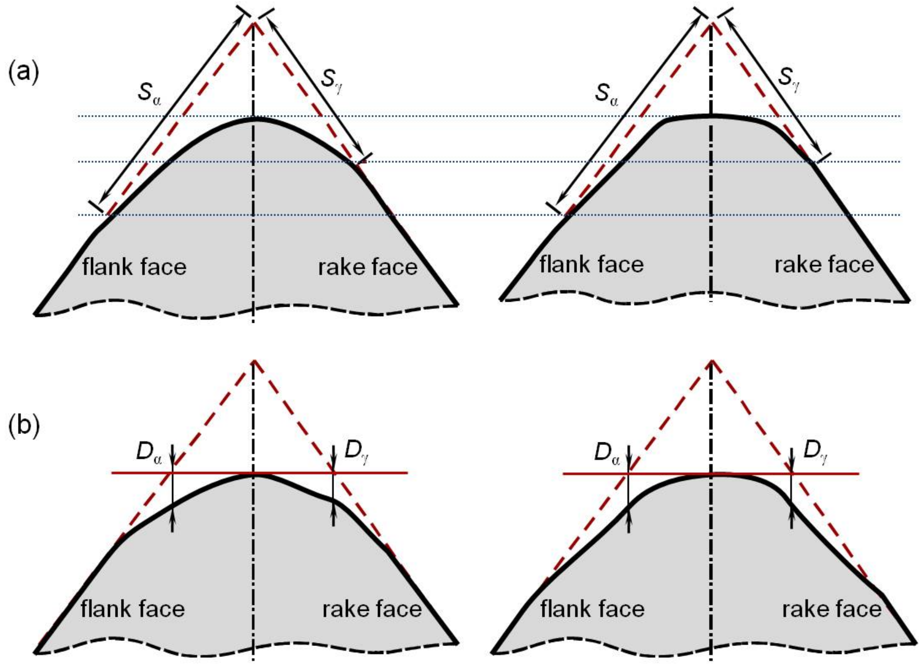

Fig. 9. Limitations of current edge characterization parameters.

curves be fit to the edge profile:

$$\begin{aligned} x(u) &= a_2u^2 + a_1u + a_0 \\ y(u) &= b_2u^2 + b_1u + b_0 \end{aligned} \tag{10}$$

Parametric quadratic curves are simple polynomials for which standard fitting algorithms are commonly available, one of which is the B-spline approximation.

In reference to the coordinate system shown in Fig. 10, the cutting edge is modeled by fitting a second degree parametric curve with $u \in [0,1]$ to the spatial coordinates of the cutting edge. The quadratic equations of the edge may be obtained directly from the parametric equations of the second segment of the tool geometry, which refers to the cutting edge profile obtained during the edge identification process (as presented in Section 2). This requires re-parametrization from $[u_1, u_2]$ to $[0,1]$ by replacing u with $[(u-u_1)/(u_2-u_1)]$. Alternatively, an additional single span approximation may be accomplished with a fixed knot ($u_i = [0,0,0,1,1,1]$) B-spline using linear algebraic algorithms. The fit of the B-spline to the cutting edge profile may be evaluated in terms of goodness-of-fit measures such as the coefficient of determination R^2,

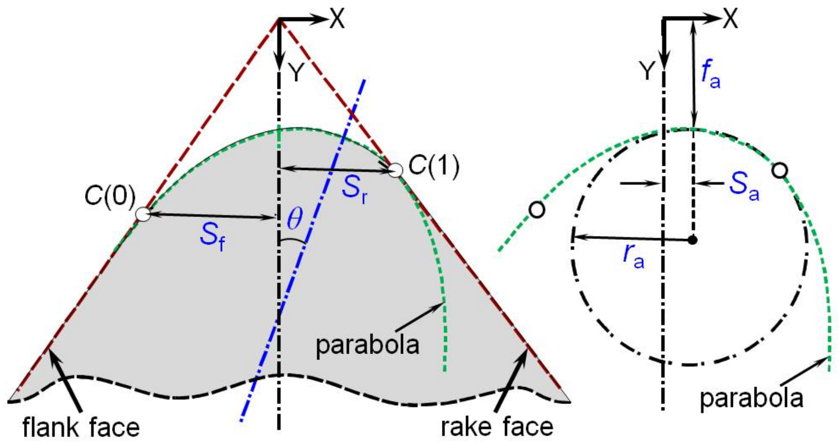

Fig. 10. Proposed edge characterization parameters.

as suggested in ref. [15].

In the interest of rendering the edge characterization parameters to comprehensible and easy to visualize, the six constants in eqn. (10) need be viewed in light of parameters that are of physical relevance. It can be shown that the implicit form of a parametric quadratic curve is a parabola [18] with the general form:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \tag{11}$$

subject to $B^2 = 4AC$. Accordingly, the minimum number of parameters to construct the edge is four, since parabolas have four degrees of freedom in the two-dimensional plane. However, to designate the cutting edge margins, two of these parameters have to correspond to the two points referring to the start and end points of the cutting edge profile. If these points are assumed to lie on the rake and flank lines, the cutting edge profile can be fully defined using four parameters, considering that for the given tool wedge angle β, $x(0) = -y(0)\tan(\beta/2)$ and $x(1) = y(1)\tan(\beta/2)$. Fig. 10 depicts these four characterization parameters to comprise two form parameters and two edge marginal parameters. If $u_a = (-b_1/2b_2)$ refers to the apex of the cutting edge, the two form parameters are:

(1) $S_a = x(u_a) = a_2(-b_1/2b_2)^2 + a_1(-b_1/2b_2) + a_0$, a measure of profile asymmetry, and
(2) $r_a = [(a_1b_2 - a_2b_1)^2/(2{b_2}^3)]$, the radius of curvature of the apex.

The two edge marginal parameters:

(3) $S_f = a_0$
(4) $S_r = a_2 + a_1 + a_0$

define the region that the edge profile extends to, with respect to the flank and rake faces, respectively. If the assumption of the start and end points of the edge profile being on the flank and rake lines is relaxed, the following two additional parameters (see Fig. 10) are required to fully define the cutting edge:

(5) $f_a = y(u_a) = (-b_1^2/4b_2) + b_0$, a measure of cutting edge flatness, and
(6) $\theta = (1/2)\tan^{-1}[(-2a_2b_2)/({b_2}^2 - {a_2}^2)]$, the angle between the axis of symmetry of the parabola and the wedge angle bisector.

The four parameters S_a, r_a, S_f and S_r can represent both symmetric and asymmetric edge profiles. S_f and S_r define the margins of the cutting edge profile and cannot therefore represent any asymmetry in the form of the edge profile. Form asymmetry is well brought out by parameter S_a that designates the location of the edge apex. Parameter r_a denotes the roundness of the edge profile.

Fig. 11 illustrates the fit of a B-spline to a symmetric edge and an asymmetric edge, the coefficient of determination R^2 for both of which exceeded 98%. For the symmetric profile (Fig. 11a), the cutting edge marginal parameters S_f and S_r are essentially the same (see Table 1). The symmetry of the edge is also denoted by the S_a and θ parameters being close to zero. If the cutting edge refers to an arc of a circle, $r_a \approx r_\beta$. For this profile, r_β was measured using circular regression over the points on the cutting edge profile to be 53 μm. For the asymmetrically honed insert shown in Fig. 11b, edge asymmetry is clearly

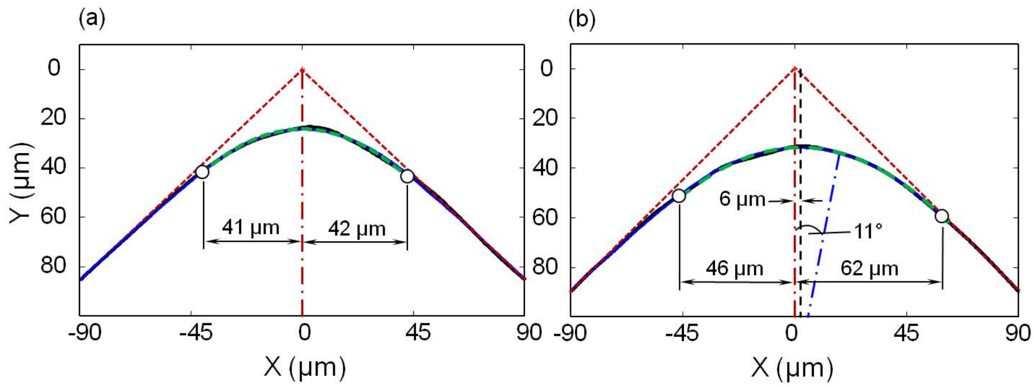

Fig. 11. Characterization of: (a) symmetric, and (b) asymmetric edges.

signified in the disparity between S_f and S_r values, and the parameters S_a and θ assuming non-zero values that are larger relative to those corresponding to the symmetric edge.

Table 1
Cutting edge characterization parameters

Parameter	$S_a(\mu\text{m})$	$r_a(\mu\text{m})$	$S_f(\mu\text{m})$	$S_r(\mu\text{m})$	$f_a(\mu\text{m})$	$\theta(°)$
Symmetric edge	0	50	41	42	25	-2
Asymmetric edge	6	65	46	62	33	11

It may be noted that the proposed characterization parameters enable the calculation of the parametric quadratic coefficients of eqn. (10), which serves to reconstruct the edge profile for further numerical analyses such as finite element modeling of cutting processes.

4 Conclusions

A parametric approach to the identification and characterization of the meso-geometry of honed cutting edges has been presented. Considering that the profile of a honed cutting edge transitions smoothly into the rake and clearance faces that flank the edge, unambiguous identification of the transition points that delimit the cutting edge from the tool faces has been demonstrated to be a critically important step that ought to precede edge characterization. Accordingly, parametric modeling of the tool edge geometry through B-splines that are presently standard modeling tools in CAD/CAM applications has been proposed.

The B-splines considered comprise three piecewise segments corresponding to the cutting edge profile and the two tool faces. The transition points that demarcate the cutting edge from the tool faces are objectively identified by the adaptive placement of the knots that minimizes the residual error referring to fitting of the B-spline to the tool profile data. This methodology has been evaluated to be robust against point uncertainty and the geometric domain over which the tool profile is modeled. Subsequent to edge identification, the edge has been modeled by parametric quadratics. The application of four geometrically-relevant, contour-based parameters derived from the best parabolic fit to the cutting edge in characterizing both symmetric and asymmetric honed edges is also demonstrated.

These parameters serve to reconstruct the cutting edge geometry for numerical analyses of cutting processes, and enable the specification of tool edge geometries that are tailored to optimize the cutting response in the manufacture of high-value components.

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